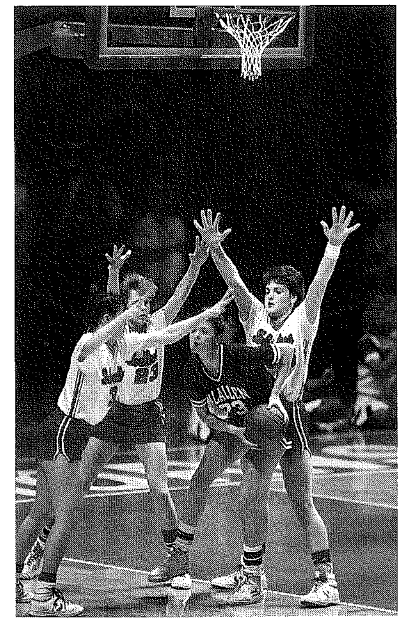
9. Suppose that your basketball team's scores in the first four games of the season were 86 points, 73 points, 76 points, and 90 points.
   a. What will your team's scoring average be if the next game score is 79 points?
   b. Write a function and draw a graph that compares the next-game score and the average score.
   c. What score will give a five-game average of 84 points?



10. Mr. and Mrs. De La Cruz want to buy a new house and must finance \$60,000 at 9.6% on the unpaid balance.
   a. Make several guesses for a monthly payment that will pay off the loan in 25 yr.
   b. Plot the points determined by your guesses (*monthly payment, balance after 25 yr*).
   c. Find a best-fit line for the points.
   d. What is the y-intercept? What is the real-world meaning of the y-intercept?
   e. What is the x-intercept? What is the real-world meaning of the x-intercept?
   f. What is the slope? What is the real-world meaning of the slope?

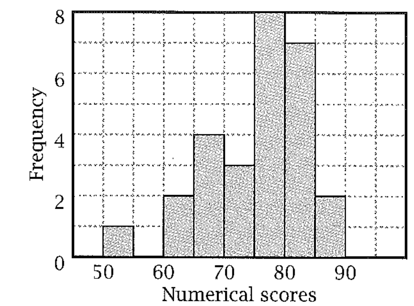
11. Use the data and the equation from Problem 8 to determine the average value of the equipment over the ten-year period. To figure this out, determine the starting value, the value each year, and the final value. Then average these numbers.

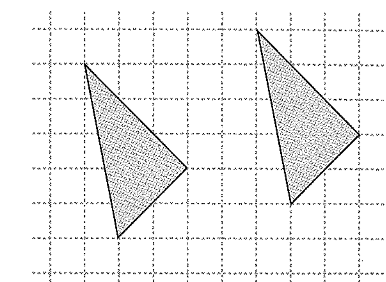
Section 5.4

The Parabola Family

The pure and simple truth is rarely pure and never simple.
—Oscar Wilde

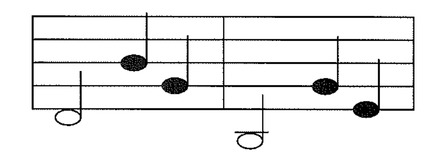
What would this graph look like if a teacher decides to add five points to each of the numerical scores pictured in the histogram below? Have any of your teachers ever done that?





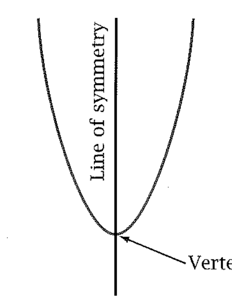
What change, or transformation, will move the triangle on the left to its position on the right? Did you do transformations like this in your geometry class?

Transformations are a natural feature of the real world, including the world of art. Music can be transposed from one key to another. Melodies are often shifted by a certain interval within a composition.



Can you name other examples of transformations?

In this section you will experiment with alterations to the equation and graph of the function pictured at the right. The **parabola** $y = x^2$ is a simple building-block function. One variation of this function models the height of a projectile from the ground as a function of time. Another variation gives the area of a square as a function of the length of a side.



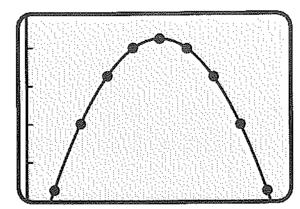
Example 1

If you are given a stick that is 100 cm in length, is there a way you can break it so that the pieces form a rectangle with the largest possible area?

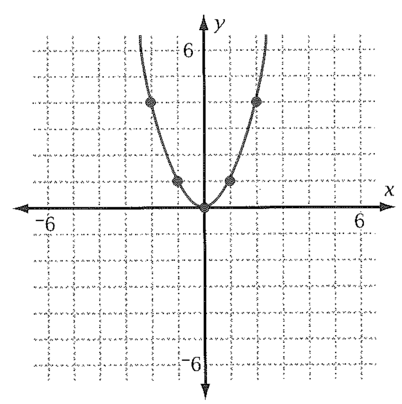
Solution

Make a table of values (x, y) = (*side length, area*) that can represent this situation. Invent several possible side lengths and then compute the associated areas.

Length (x)	5	10	15	20	25	30	35	40	45
Width	45	40	35	30	25	20	15	10	5
Area (y)	225	400	525	600	625	600	525	400	225



The graph of (*side length, area*) is a parabola that is upside-down and translated from its usual position. Do you see that the **vertex** is at the highest point and shows that a side length of 25 cm gives the greatest area, which is 625 cm^2? For this example, what geometric shape has maximum area? ■



Set a "friendly" graphing window in which the distance between the pixels is 0.2 on both the x- and y-axes. Enter this y-symmetric graph on your calculator as $y = x^2$. Some of the points not shown on the screen include $(^-3, 9)$, $(3, 9)$ and $(^-4, 16)$, $(4, 16)$.

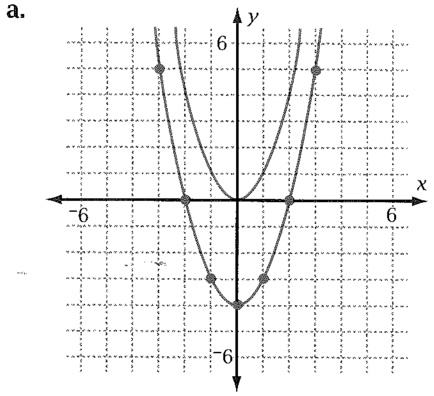
All parabolas are related to this simple parent function: $y = x^2$ with the **vertex** at $(0, 0)$. The major focus of this section will be for you to (a) slide, or translate, this graph to a new position, (b) write the equation of a parabola based on its shape and the position of its vertex, and (c) predict the graph of a parabola given its equation. Locating the vertex and connecting it to an equation is your key to success with parabolas.

Investigation 5.4.1

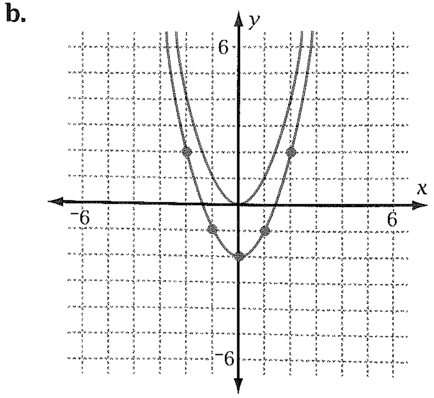
Make My Graph

The function $y = x^2$ is shown on each graph below. Experiment by changing this equation to find out how you can slide the parabola up and down. Use a combination of calculator guess-and-check, logic, and common sense. Try to learn from your mistakes. When you have succeeded, write the equation that worked.

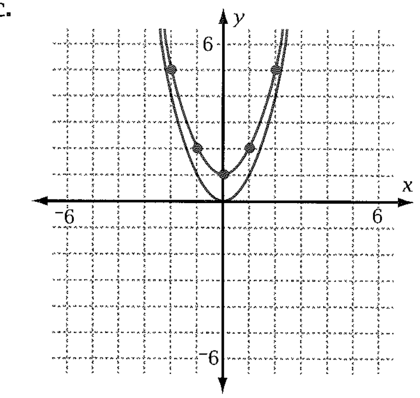
a.



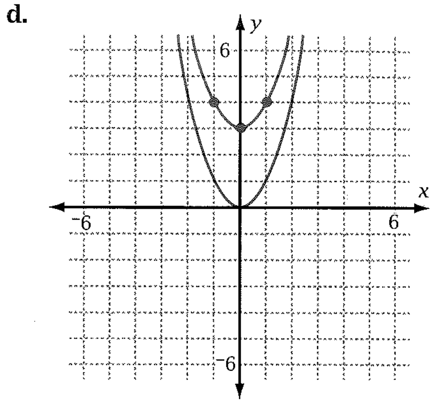
b.



c.



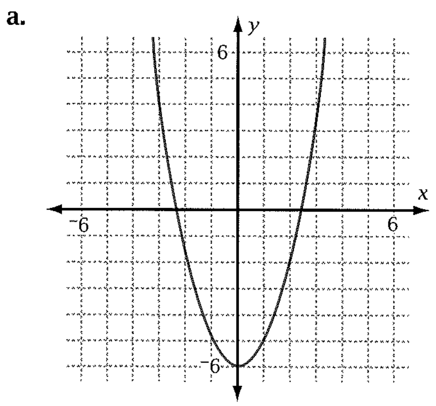
d.



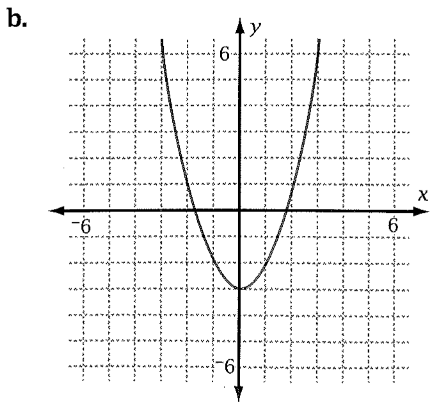
Problem Set 5.4

1. Use your calculator to find the equation for each parabola. Each parabola is congruent to $y = x^2$.

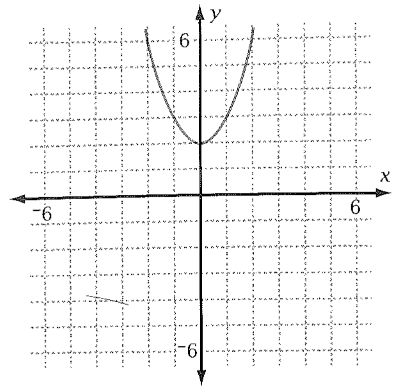
a.



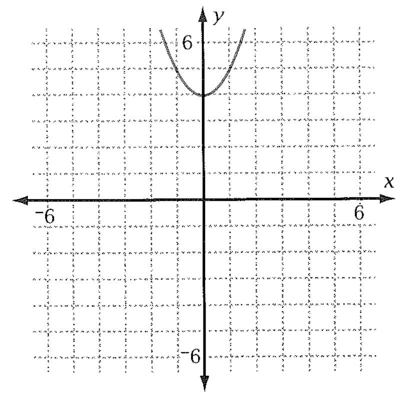
b.



c.



d.



2. In each part of this problem, the graph will be a parabola that is congruent to the parent function $f(x) = x^2$.

a. Write the equation and draw the graph of a parabola that is congruent to $f(x) = x^2$ with the given translation.

i. six units down from $f(x)$ ii. two units up from $f(x)$

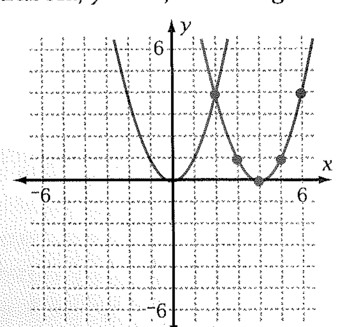
b. Write an equation in $y=$ form and describe the location of each parabola relative to $f(x)$.

i. $f(x) - 5$ ii. $f(x) + 4$

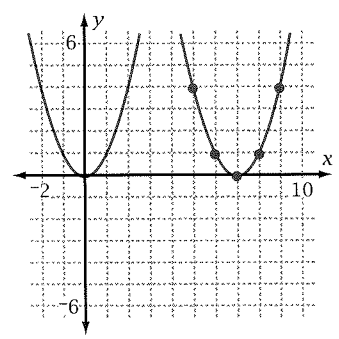
c. In general, $y = x^2 + c$ means the same thing as $f(x) + c$ when $f(x) = x^2$. Describe the location of $f(x) + c$.

3. Now that you have discovered how to move the parabola up and down, experiment to find out how to move it from side to side. Write the equation you used to slide the parabola, $y = x^2$, to the right or left.

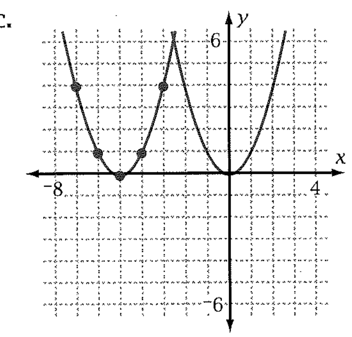
a.



b.



c.



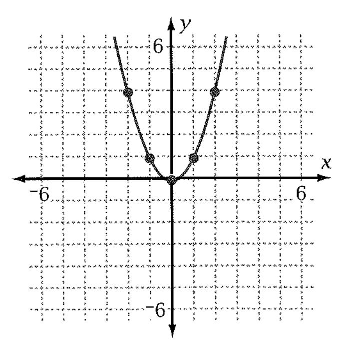
4. Describe what happens to the graph of $y = x^2$, pictured at right, in the following situations.

a. x is replaced by $(x - 3)$.

b. x is replaced by $(x + 3)$.

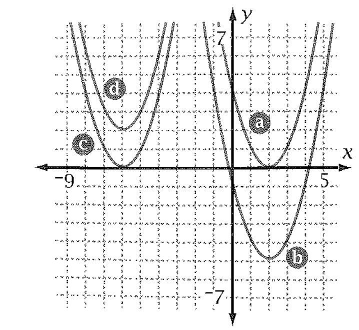
c. y is replaced by $(y - 2)$.

d. y is replaced by $(y + 2)$.



5. Write an equation for each parabola pictured at the right. (See **Calculator Note 5C** for additional practice.)

NOTE
5C



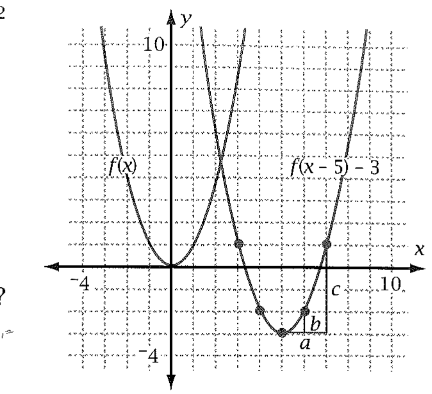
6. a. Describe a two-step process for sliding $f(x) = x^2$ to the location of $f(x - 5) - 3$, as pictured at the right.

b. Write the equation of this parabola in $y=$ form.

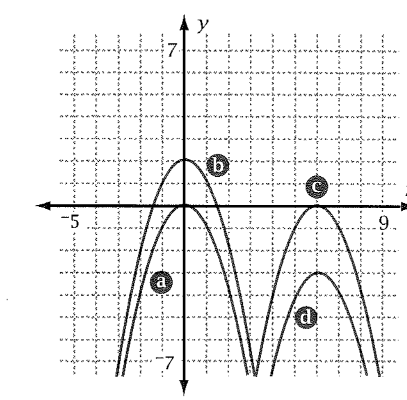
c. Where is the vertex of this parabola?

d. What are the coordinates of the other four points if they are one and two horizontal units from the vertex?

e. What is the length of segment b? Of segment c?



7. Write the equation and graph each parabola on your calculator. (Each parabola is congruent to $y = x^2$.)



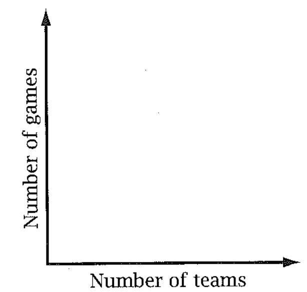
8. Write the equation of a graph that is congruent to $y = {}^-1(x + 3)^2 + 4$, but shifted five units right and two units down.

9. The **graphing form** of the equation of a parabola is often written as $y = \pm 1(x - h)^2 + k$. Based on your work in the previous problems, explain the effect of ± 1, h, and k on the graph of the parent function. Be as explicit as possible.

10. Write an equation of the form $y = \pm 1(x - h)^2 + k$ that provides the table values for the (*length, area*) data of Example 1 in this section. Graph your equation to check your answer.

11. Make a table of values that compares the number of teams in a league and the number of games required for each team to play every other team twice (once at home and once away from home).

Number of teams	1	2	3	...	10
Number of games	0	2	6	...	-?-



Plot each point and describe the graph produced. Write an explicit function for this graph.

12. The height of a bowling ball dropped from the top of a 64-foot tower is given by $h = {}^{-}16t^2 + 64$, where t is the number of seconds after the ball has been dropped, and h is the height of the ball in feet. The questions here will look at the average height of the ball during the first two seconds.

a. Make a table showing the time and the height of the ball when it is dropped, when it hits the ground, and every $\frac{1}{2}$ sec in between. Then average these heights.

b. Find the average height by using the heights every $\frac{1}{4}$ sec.

c. Find the average height by using the heights every $\frac{1}{10}$ sec.

d. Explain the differences in your answers to 12a, b, and c. Which best answers the question, "What is the average height?" Why?

Take Another Look 5.4

How familiar are you with the Periodic Table of Elements? Have you or will you soon take a chemistry course? Make a scatter-plot of (*atomic number, atomic mass*) for the first 12 elements listed. Find the least-squares model for this information. Find the residuals for the first 12 elements using this model. Then use your model to predict the atomic mass of mercury, copper, lead, silver, and gold. Find the residuals for these five elements and discuss the accuracy of this model. Ask a chemistry teacher or chemistry-aware friends to help you find the information you need.

Project

Even and Odd Functions

Graph each of the functions $y = x^2$, $y = \sqrt{4 - x^2}$, and $y = |x|$. Describe any symmetry appearing in these graphs. These functions are examples of **even functions**. A function f is even when $f({}^{-}x) = f(x)$ for all defined values of x. Explain how this definition relates to the symmetry you see in the graphs.

Graph these examples of **odd functions**: $y = x^3$, $y = \frac{1}{x}$, and $y = \sqrt[3]{x}$. Now rotate your calculator 180°. Each graph will look the same as it looked before the rotation. This property is called symmetry with respect to the origin. Odd functions are defined as those functions f where ${}^{-}f(x) = f({}^{-}x)$ for all defined values of x. Explain how this definition relates to the symmetry you see in the graphs.

Give an example of a function that is neither even nor odd. Graph your example and explain why you think it qualifies.

When functions are combined, the symmetry may change. Use functions from the examples given for even and odd functions above to investigate what happens when even and odd functions are combined. Is the result even, odd, or neither? Make a table like the one below and record your results. The notation $f(g(x))$, which is used to combine functions, may be unfamiliar to you. If so, be sure to read Section 5.9 for an explanation.

	$f(g(x))$	$g(f(x))$	$f(x) + g(x)$	$f(x)g(x)$	$\frac{f(x)}{g(x)}$
$f(x)$ even, $g(x)$ even	-?-	-?-	-?-	-?-	-?-
$f(x)$ even, $g(x)$ odd	-?-	-?-	-?-	-?-	-?-
$f(x)$ odd, $g(x)$ odd	-?-	-?-	-?-	-?-	-?-

Choose three entries from the table and prove your result. A sample proof is given below.

Prove: If $f(x)$ is even and $g(x)$ is odd, $f(g(x))$ is even.

If $g(x)$ is odd, then $g({}^{-}x) = {}^{-}g(x)$. This means $f(g({}^{-}x)) = f({}^{-}g(x))$.

If $f(x)$ is even, then $f({}^{-}g(x)) = f(g(x))$.

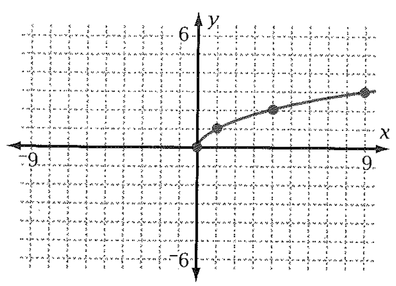
Therefore, $f(g({}^{-}x)) = f(g(x))$ and hence $f(g(x))$ is even.

Section 5.5

The Square Root Family

The pursuit of pretty formulas and neat theorems can no doubt quickly degenerate into a silly vice, but so can the quest for austere generalities which are so very general indeed that they are incapable of application to any particular.
—Eric Temple Bell

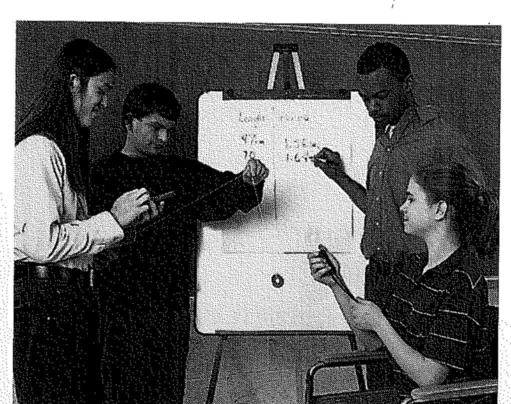
The square root graph is another parent function that can be used to illustrate transformations. Both the domain and range of $f(x) = \sqrt{x}$ are real numbers that are zero or greater. If you trace the graph, there are no function values for y unless x is at least zero. Trace to show that $\sqrt{3}$ is approximately 1.732 and that $\sqrt{8}$ is approximately 2.828. Describe how you would use the graph to find $\sqrt{31}$. What happens when you try to trace for values of $x < 0$? What is $\sqrt{^-4}$?

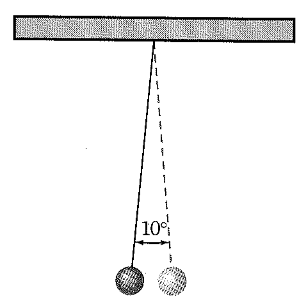


Investigation 5.5.1

Pendulum Experiment

For this investigation, the group will need some string and an object to use as a weight. You will also need a stopwatch or a watch with a second hand.





a. Tie the weight at one end of a length of string. Firmly hold the other end of the string or tie it to something, so that the weight hangs freely.

b. Measure the length from the center of the weight to the point where the string is held.

c. Carefully extend and release the weight so that it swings back and forth in a short arc of about 10°. Time ten complete swings; forward and back is one swing.

d. Find the period by dividing by ten. The period of your pendulum is the time for one complete swing (forward and back).

e. Repeat the experiment for several different string lengths and complete a table of values like the one below. Use a variety of short, medium, and long string lengths. Save these data for one of the problems in Problem Set 5.5.

Length (cm)	-?-	-?-	-?-	-?-	-?-	-?-
Period	-?-	-?-	-?-	-?-	-?-	-?-

You can use the square root function when finding the height of a falling object. Example 1 shows you how to do this.

Example 1

An object falls to the ground because of the influence of gravity. When an object is dropped from a height of d meters, the height after t seconds is given by $h(t) = {}^-4.9t^2 + d$. If an object is dropped from a height of 1000 meters, the height of the object at any given time (in seconds) is given by the function $h(t) = {}^-4.9t^2 + 1000$. How long does it take for the object to reach the ground?

Solution

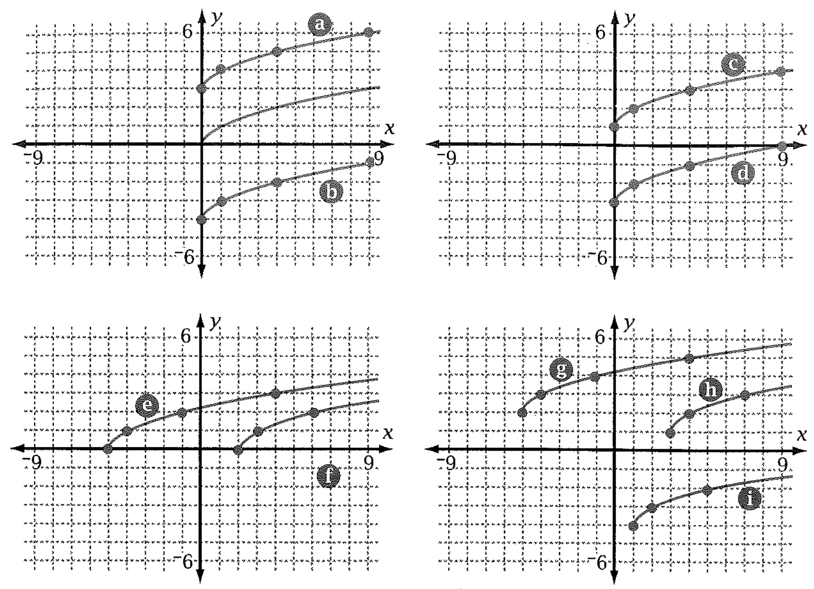
You are finding the time until the height will be 0.

$0 = {}^-4.9t^2 + 1000$	Set the height = 0.
$4.9t^2 = 1000$	Add $4.9t^2$ to both sides.
$t^2 = \frac{1000}{4.9}$	Divide both sides by 4.9.
$t = \sqrt{\frac{1000}{4.9}} \approx 14.3$ seconds	Take the square root of both sides. ■

Certainly the falling time depends on the starting height. How long will it be until an object reaches the ground, if it is dropped from 800 meters? From 650 meters? From d meters? This is just one of many functional relationships involving the square root function.

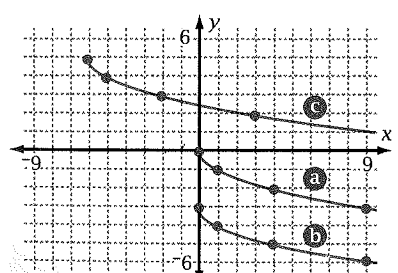
Problem Set 5.5

1. Write the equation for each transformed graph. The parent function is $y = \sqrt{x}$. Verify each answer by graphing it on your calculator.



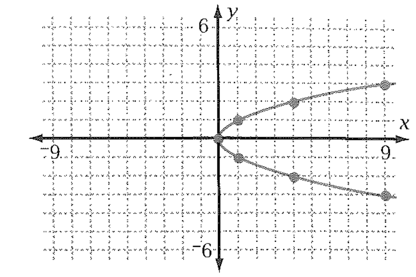
2. a. What happens to the graph $y = \sqrt{x}$ if x is replaced with $(x - 3)$? With $(x + 3)$?
   b. What happens to the graph $y = \sqrt{x}$ if y is replaced with $(y - 2)$? With $(y + 2)$?

3. Write an equation for each of the three graphs below.

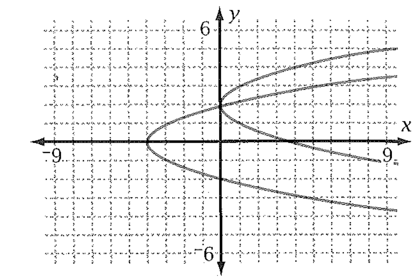


4. Consider the parent function $f(x) = \sqrt{x}$.
   a. Name three pairs of integer coordinates that are on the graph of $f(x + 4) - 2$.
   b. Write $f(x + 4) - 2$ in $y=$ form and graph it.
   c. Write $^{-}f(x - 2) + 3$ in $y=$ form and graph it.

5. a. Graph this parabola (shown at right) on your own calculator. (Hint: You will need to graph two functions.)
   b. Combine the two functions in 5a and write the equation in condensed form, $y = \pm\sqrt{x}$. When both sides of this equation are squared, what is the resulting equation?



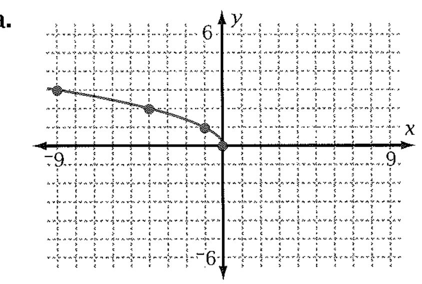
6. Refer to the two parabolas at the right.
   a. Explain why neither graph represents a function.
   b. Write equations for each parabola in the condensed form $y = \pm$(a square root expression).
   c. Square both sides of each equation in 6b. What is the resulting equation for each parabola?



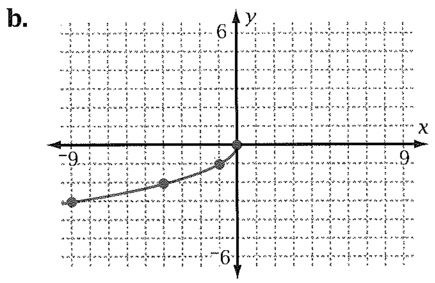
7. Graph the parabola $(y - 2)^2 = x + 3$.

8. Write the equation for each graph.

a.



b.



9. a. Enter the data collected during Investigation 5.5.1 into your calculator.
   b. Plot the data, and write an equation that best fits the data. Graph the residuals.
   c. Use your model to predict the period for a 10-meter pendulum length.
   d. Find the length for a clock pendulum that has a period of one second.

10. The following table provides information about the (*distance, velocity*) relationship for an object dropped near the earth's surface. The distance the object has fallen is measured in feet, and velocity is in feet per second. Use guess-and-check to write a square root function that provides a good fit for the data.

Distance (ft)	0	2.5	5	10	15	20	25
Velocity (ft/sec)	0	12.65	17.89	25.30	30.98	35.78	40.00

11. Using **Calculator Note 5C** as a guide, write a program that will randomly generate graphs of square root functions. Give a line-by-line explanation describing how the program works.

NOTE
5C

Take Another Look 5.5

Example 1 in this section describes the height of a falling object dropped from a height of 1000 meters. You will now use similar functions to describe a ball that is thrown vertically at 32 m/sec. How high do you think it will get before falling back to the ground? How long will it take before the ball hits the ground? Make two sketches that show (*time, height*) and (*time, velocity*) of the thrown ball. (The velocity of the moving ball is the ratio $\frac{\text{difference in height}}{\text{difference in time}}$ for two different positions. Positive, zero, and negative velocity values can help you interpret the ball's movement.)

a. The functions $h(t) = {}^-4.9t^2 + 32t + 2.1$ and $v(t) = {}^-9.8t + 32$ can be used to predict the height and velocity at any time while a thrown object is moving. Graph the two functions simultaneously.

Time (sec)	Height (m)	Velocity (m/sec)
0	-?-	-?-
1	-?-	-?-
2	-?-	-?-
3	-?-	-?-
4	-?-	-?-
5	-?-	-?-
6	-?-	-?-
7	-?-	-?-

i. Complete a table like the one above, giving the height and velocity for each time listed.

ii. Find the maximum height of the ball and the time it takes to reach that height. What is the velocity when the ball is at its maximum height? Describe the velocity before and after that time.

iii. Explain the relationship between the two graphs before the maximum height, at the maximum height, and after the maximum height.

iv. Describe realistic domains and ranges for the two functions.

v. In the context of this activity, describe the real-world meaning of every variable, number, operation, and expression term in each of the two functions provided.

b. Find the velocity of the ball at the time it hit the ground and explain how you determined this.

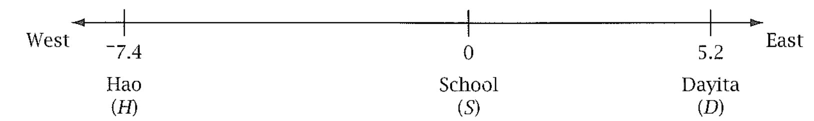
c. The graph of the (*time, height*) relationship is symmetrical. Carefully explain the height and velocity implications of this symmetry.

Section 5.6

The Absolute Value Family

The test of a first-rate intelligence is the ability to hold two opposed ideas in the mind at the same time, and still retain the ability to function.
—F. Scott Fitzgerald

Hao and Dayita ride the subway to school each day. They live on the same east-west subway route. Hao lives 7.4 miles west of the school, and Dayita lives 5.2 miles east of the school. This information is shown on the number line below.

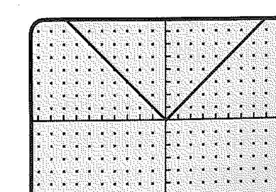


The distance from Hao's stop to school, *HS*, is 7.4 units. The distance from Hao's stop to Dayita's stop, *HD*, is 12.6 units. When you talk about distance, you usually don't mention direction, unless you're interested in the directed distance. Therefore, a distance will be either positive or zero.

This is exactly what the **absolute value** function does. It makes numbers positive or zero. The job is easy when the number is zero or a positive number, and not difficult when the number is negative. The mathematical notation for the absolute value of $^-3$, or the distance from $^-3$ to the origin, is $|{}^-3|$. What is the value of $|{}^-3|$? What is the value of $|5.2 - {}^-7.4|$, the distance from *D* to *H*? What is the value of $|13.4|$?

The absolute value function was first described by the French mathematician Augustin-Louis Cauchy in the 1820s. The symbol used today for absolute value was introduced by the German mathematician Karl Weierstrass in 1841.

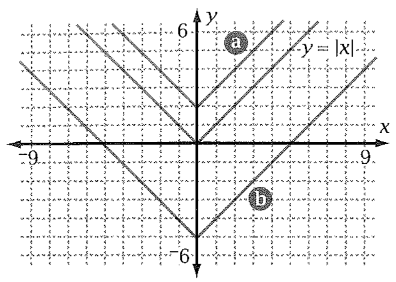
When you determined the absolute deviation, or the distance from a data point to the mean, you were using the absolute value function. You also used this function when finding the mean absolute value of the residuals.

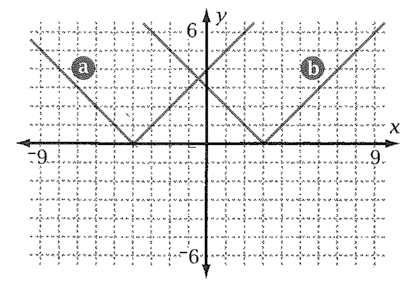


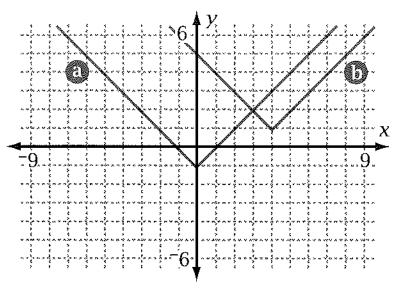
In this section you will learn about the graph of $y = |x|$ as another example of a function that can be transformed. The parent function $y = |x|$ is shown at the right. Graph this equation on your calculator. In the upcoming problems, you will find equations of the form $y = a|x - h| + k$. What you have learned about moving other graphs will work with this function as well.

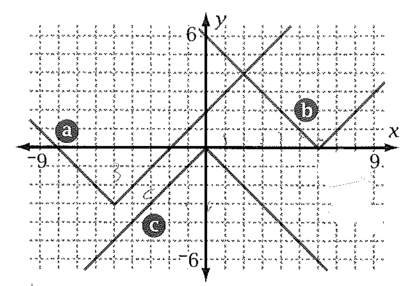
Problem Set 5.6

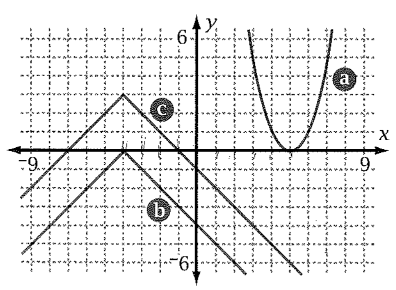
In Problems 1-6, write the equation of each graph. Be sure to check each equation by graphing it on your calculator.

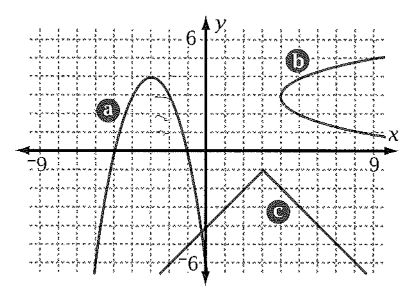
1.


2.


3.


4.


5.


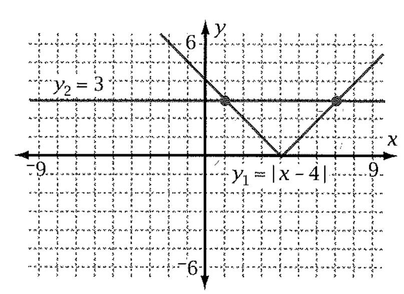
6.


7. a. If $f(x) = |x|$, what is the equation in $y=$ form for $^-f(x - 2) + 3$?
   b. Describe the transformations needed to move $f(x)$ into this position.

8. a. What is the graphic result when x is replaced by $(x - 5)$ in an equation?
   b. What is the graphic result when x is replaced by ^-x in an equation?
   c. What is the graphic result when y is replaced by $(y - 3)$ in an equation?
   d. What is the graphic result when y is replaced by ^-y in an equation?

9. At the right is an illustration of how to solve the equation $|x - 4| = 3$ graphically. The equations $y_1 = |x - 4|$ and $y_2 = 3$ were graphed on the same coordinate axes.

   What is the x-coordinate of each point of intersection? What x-values are solutions of the equation $|x - 4| = 3$?



10. Solve the equation $|x + 3| = 5$ using the method shown in Problem 9. Sketch the graph on your paper, and indicate where the solutions are on the graph.

11. Graph two equations that show the x-value(s) when $^-(x + 3)^2 + 5 = |x - 1| - 4$. Sketch a graph on your paper and indicate where the solutions are on your graph. Trace and zoom to find these values to the nearest 0.1.

12. Sketch the graphs of $y = x$, $y = {}^-x$, and $y = |x|$ on the same axes. Use a different colored pen or pencil for each equation. Study the three graphs, especially where they overlap, and write a definition of $|x|$ in terms of x and ^-x.

13. Each year the local school district schedules a mathematics and science fair. A panel of judges rates each exhibit. The ratings for the top 20 exhibits are shown in the table.

Exhibit	1	2	3	4	5	6	7	8	9	10
Rating	68	71	73	77	79	79	81	83	83	84

Exhibit	11	12	13	14	15	16	17	18	19	20
Rating	85	86	88	89	89	90	92	92	92	94

The judges decide that the top rating should be 100, so they add 6 points to each rating score.

   a. What was the mean and the mean absolute deviation of the ratings before raising them? After raising them? What do you notice about the change in the mean? In the mean absolute deviation?
   b. Plot the original ratings, using the exhibit number as the x-coordinate.
   c. Plot the raised ratings on the same graph. Describe how the alteration of the ratings affected the graph.

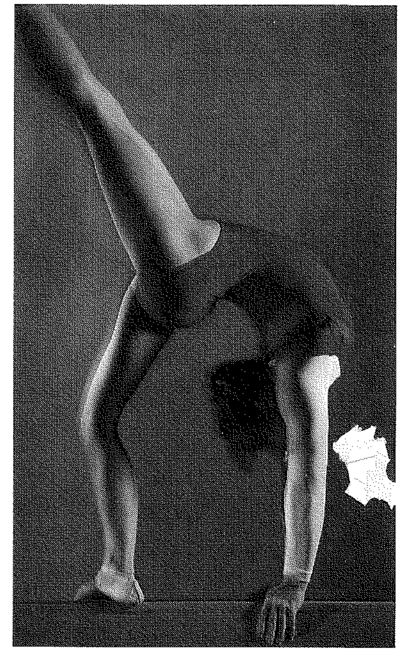
14. You can use a single receiver to find the distance to a homing transmitter by measuring the strength of the signal, but you cannot determine the direction from which the signal is emanating. The following distances were measured while driving east along a straight road. Find a model that closely fits the data. Where do you think the homing transmitter might be located?

Miles traveled	0	4	8	12	16	20	24	28	32	36
Distance to object	18.4	14.4	10.5	6.6	3.0	2.6	6.0	9.9	13.8	17.8

Section 5.7

Stretching a Curve

My method to overcome a difficulty is to go around it.
—George Polya

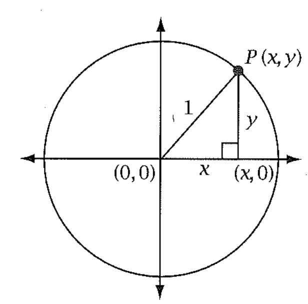


You have explored several functions and moved them around the plane. You know that a horizontal translation occurs when x is replaced by $(x - h)$, and a vertical translation occurs when y is replaced by $(y - k)$. You have flipped, or reflected, graphs over the y-axis and x-axis by replacing x with (^{-}x), and y with (^{-}y), respectively. In each case, however, the final image was the same size and shape as (or congruent to) the original graph.

You can also distort a graph so that the image isn't congruent to the original graph. One of the easiest ways to do this is to stretch the y-values, or the x-values, or both. In Section 5.5 you stretched the graph of $y = \sqrt{x}$ to fit the (*distance, velocity*) relationship for an object dropped near the earth's surface. Using guess-and-check, you were able to show that $v = 8\sqrt{d}$ provides a good fit with the data.

Distance (ft)	0	2.5	5	10	15	20	25
Velocity (ft/sec)	0	12.65	17.89	25.30	30.98	35.78	40.00

The graph that shows distortions most clearly is the circle. Suppose P is a point on a unit circle with center at the origin. A **unit circle** has a radius of one unit.



You can derive the equation of a circle from this diagram by using the Pythagorean theorem. The equation is $x^2 + y^2 = 1$, because the legs of the right triangle are of lengths x and y and the length of the hypotenuse is one unit.

Equation of a Unit Circle

$x^2 + y^2 = 1$ is the equation of a **unit circle** with center (0, 0).

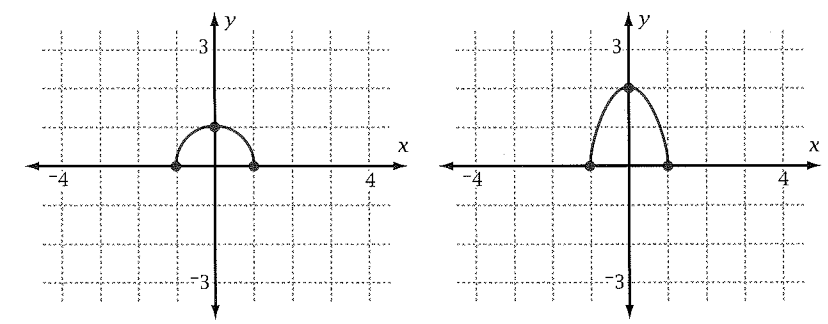
What is the domain and range of this circle? If a value, like 0.5, is substituted for x, what are the output values of y? Is the graph a function? Why or why not?

In order to draw the graph of a circle on your calculator, you will need to solve the above equation for y. When you do this, you will get two equations, $y = {}^{+}\sqrt{1 - x^2}$ and $y = {}^{-}\sqrt{1 - x^2}$. By graphing both of these equations, you will be able to draw the complete circle. Be sure to use a "friendly" graphing window so that the circle will look like a circle and not an ellipse.

Problem Set 5.7

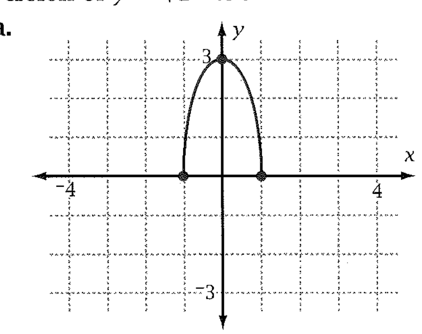
When possible, verify your work by graphing each equation on your calculator. Be sure to use a "friendly" graphing window.

1. The equation $y = \sqrt{1 - x^2}$ is the equation for the top half of the unit circle with center at (0, 0) shown on the left. Alter this equation to graph the figure on the right below. What is the equation of the semicircle after it has been stretched vertically?

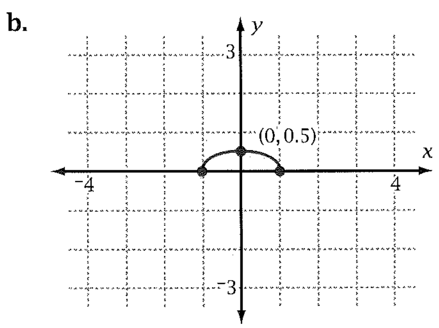


2. Write the equation of each graph. The graph will be a **stretched** or **compressed** version of $y = \sqrt{1 - x^2}$.

a.



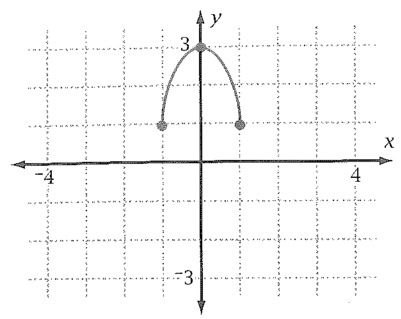
b.



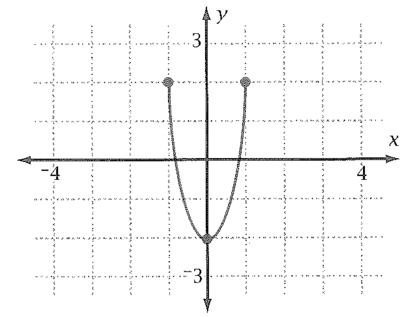
3. Consider the function $f(x) = \sqrt{1 - x^2}$, and graph each of the functions below.
   a. $^{-}f(x)$
   b. $^{-}2f(x)$
   c. $2f(x) - 3$

4. Write an equation in $y=$ form for each graph.

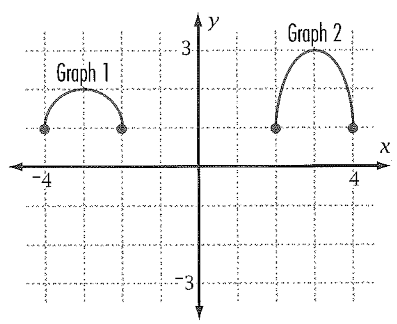
a.



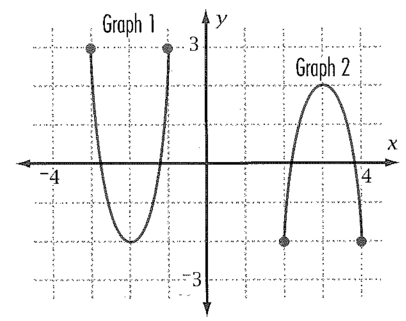
b.



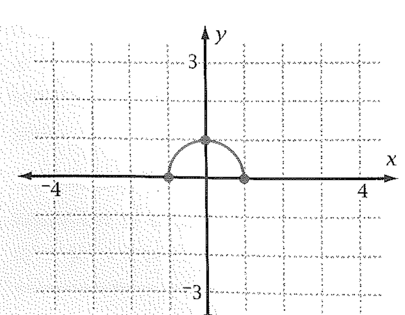
5. Suppose the top half of the unit circle with center (0, 0) is named $f(x)$.
   a. Write the equation of each graph below in terms of $f(x)$.
   b. Write the $y=$ form of each graph.



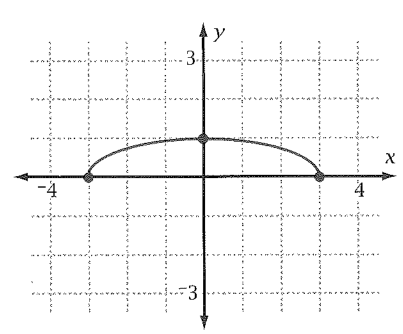
6. Write an equation in $y=$ form for each graph below.



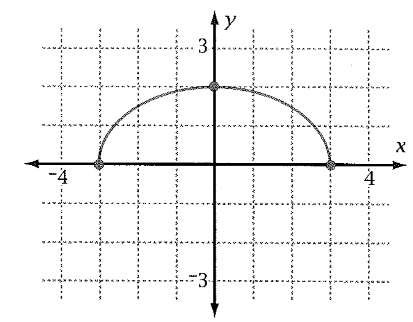
7. Write an equation, and graph each distortion of the unit semicircle.
   a. Replace y by $(y - 2)$.
   b. Replace x by $(x + 3)$.
   c. Replace y by $\frac{y}{2}$.
   d. Replace x by $\frac{x}{2}$.



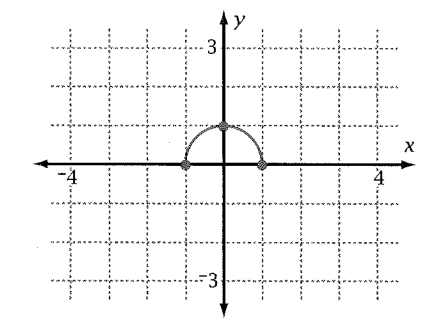
8. a. Write the equation of the distorted semicircle shown below. The x-coordinate of each point has been stretched by a factor of three.
   b. What term did you use to replace x in the parent equation?
   c. If $f(x)$ was the original semicircle, then what is this new function in $f(x)$ notation?



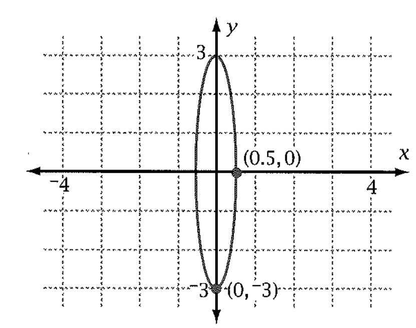
9. In the figure below, the y-coordinate of each point of the semicircle has been stretched by a factor of two, and the x-coordinate has been stretched by a factor of three.
   a. What is the equation of this set of points?
   b. What is the equation of the reflection image over the x-axis?



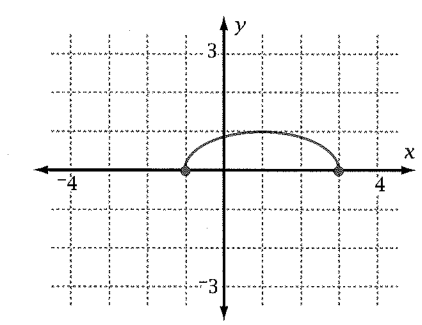
10. Given the semicircle pictured below, write the equation that generates each distortion.
    a. Each y-value is half the original y-value.
    b. Each x-value is half the original x-value.
    c. Each y-value is half the original y-value, and each x-value is twice the original x-value.



11. a. Write two equations that can be used to graph this relation.
    b. Write one equation in $y = \pm$ form that could be used to replace the two equations in 11a.
    c. Write another equation by squaring both sides of the equation in 11b.



12. Write the equation of the transformed semicircle, pictured below, in the following forms.
    a. In $y=$ form
    b. In $f(x)$ form



13. Refer to Problem Set 5.6, Problem 13. After the judges raised the rating scores by adding the same number to each score, one of the judges suggested that perhaps they should have multiplied the original scores by a factor that would make the highest score equal 100. They decided to try this method.

Exhibit	1	2	3	4	5	6	7	8	9	10
Rating	68	71	73	77	79	79	81	83	83	84

Exhibit	11	12	13	14	15	16	17	18	19	20
Rating	85	86	88	89	89	90	92	92	92	94

a. By what factor should they multiply the highest score, 94, to get 100?
b. Use this same multiplier to alter all of the scores, and record the altered scores.
c. What is the mean and the mean absolute deviation of the original scores? Of the altered scores?
d. Plot the original and altered scores on the same graph. Describe what happened to the scores visually. How does this explain what happened to the mean and the mean absolute deviation?
e. Which method do you think the judges should use? Explain your reasoning.

14. What is the average value of the function $y = \sqrt{1 - x^2}$ in the interval between $x = {}^-1$ and $x = 1$?
a. Find the value of the function at $^-1$, $^-0.5$, 0, 0.5, and 1. Then average these values.
b. Find the value of the function at $^-1$, $^-0.8$, $^-0.6$, . . . , 0.8, and 1. Then average these values.
c. Repeat this process once more using more closely spaced x-values.
d. Compare the answers from 14a, b, and c. If you continue this process with more and more closely spaced x-values, what would you expect to get for an average value? You can use a program that will compute the average value of a function. (See **Calculator Note 5D.**)
e. Give a line-by-line explanation of how the average value program works.

NOTE **5D**

15. You probably used your graphing calculator quite a bit in this chapter to explore several families of graphs. Some people might think it isn't necessary to study these families because graphing calculators are available. What do you think? Support your opinion with clear statements and examples.

Project

The Greatest Integer Function

In this chapter you have studied in detail five parent functions and several generic functions. You have shifted, flipped, and stretched functions. In this project you will look at a quite different, but important, parent function known as the greatest integer function. Its full name is the "Greatest Integer Less Than or Equal To Function." The symbol $[\![x]\!]$ is used for this function, but most computers and calculators use int(x) in the same way abs(x) is used for absolute value. Change the calculator mode to *disconnected* or *dot graphs* to study this function, because the function is not always smooth and it jumps at times. (See **Calculator Note 5E** for specific notes on your calculator.)

NOTE **5E**

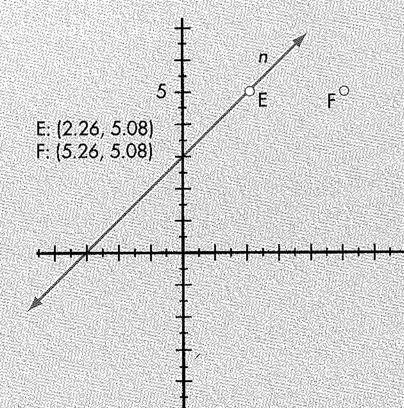
Your task is to explain what the function does, to describe its graph with both words and pictures, and to give a detailed account using complete sentences of how this function behaves with shifts, flips, and stretches (both vertical and horizontal). Look at both the overall nature of the function and at what happens at specific x-values, such as 2.5, 4.7, $^-3.1$, 5, $^-4$, and so on.

Geometer's Sketchpad Investigation

Transforming a Point and a Line

It is sometimes difficult to see exactly what happens when you apply transformations to a line. However, by using Sketchpad and by looking at a point on the line and then tracing its locus, you can better visualize these transformations.

Part 1 In Part 1 of this investigation you will explore how translations affect the equation and the graph of a line.

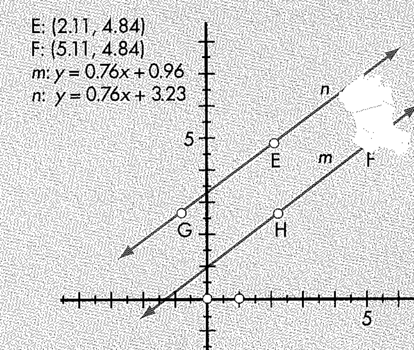


Step 1 Choose Create Axes from the Graph menu, and draw line n anywhere.

Step 2 Construct point E on line n and measure its coordinates.

Step 3 Translate point E horizontally 3 cm. Label this new point F. Measure the coordinates of point F.

Step 4 Select point F and choose Trace Point. Move point E along line n and observe the path of point F.



Step 5 Construct another point on line n and label this point G. Translate point G to point H by using the same transformation that was used in Step 3.

Step 6 Construct a line through point F and point H. Label this line m.

Step 7 Measure the equations of line n and line m.

Questions

1. Observe how the coordinates of point F change with respect to the coordinates of point E. Describe the locus of point F with respect to point E.
2. Compare the equations for line n and line m. How did the transformation affect the slope and the y-intercept of the original line?
3. What happens as you rotate the original line? When does the translated line become concurrent with the original line?
4. Experiment with other horizontal and vertical translation components. What happens when you do both a horizontal and a vertical transformation at the same time? Summarize your discoveries.

Part 2 In this part of the investigation you will see how stretches affect the equation and the graph of a line.

Step 1 Start with a new sketch. Choose Create Axes from the Graph menu, and draw line n.

Step 2 Construct point E on line n and measure its coordinates.

Step 3 Construct a line through point E that is perpendicular to the x-axis. Construct the point of intersection of this line with the x-axis. Label this point F.

Step 4 Mark point F as a center, and dilate point E around this center by 60%. Label this dilated point G. Measure the coordinates of point G. Hide the perpendicular line and point F.

Step 5 Select point G and choose Trace Point. Move point E along line n and observe the path of point G.

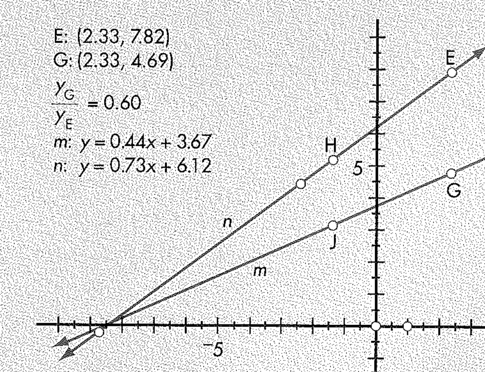
Step 6 Calculate the ratio of the y-coordinate of point G to the y-coordinate of point E.

Step 7 Construct another point on line n and label this point H. Construct a line through point H that is perpendicular to the x-axis. Construct the point of intersection of this line with the x-axis. Label this point K.

Step 8 Mark point K as a center and dilate point H around this center by 60%. Label this dilated point J. Measure the coordinates of point J. Hide the perpendicular line and point K.

Step 9 Construct a line through point G and point J. Label this line m.

Step 10 Measure the equations of line n and line m.



Questions

1. Try dynamically changing the position and slope of line n. Does the relationship between the equations of the two lines stay the same? What is the relationship?
2. Unless the two lines are parallel to the x-axis, both of the lines intersect at a point where $y = 0$. Use algebra to show why.
3. Open a new sketch and do a similar construction to show the effect of a dilation on the x-coordinate of a point. Explore what happens if you apply a horizontal stretch on a line.

Section 5.8

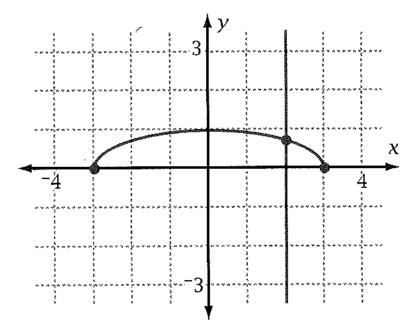
A Summary

When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again. . . .
—Isidore of Seville

In this chapter, you have studied a variety of graphs. You have learned how to recognize a graph and match it with its basic, or parent, equation.

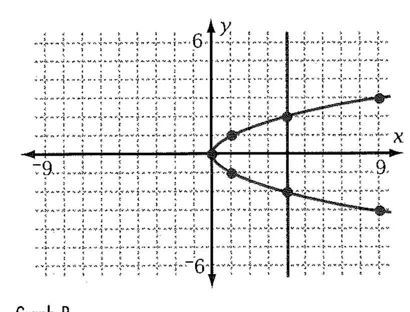
Graph	Parent equation
line	$y = mx + b$
square root	$y = \sqrt{x}$
absolute value	$y = \lvert x\rvert$
parabola	$y = x^2$
semicircle	$y = \sqrt{1 - x^2}$

A **function** is a relationship between two variables in which there is exactly one value of the dependent variable (y) for each value of the independent variable (x). You see functions as graphs, equations, two-variable data, or verbal descriptions. Function equations can be defined recursively or explicitly. The data might be in the form of points (x, y) or tables of information. But in each instance where y is a **function** of x, exactly one output value is paired with each input value. Graph A displays this quality. If you draw a vertical line at any x-value, it will not intersect the graph at more than one point. This is called the **vertical line test** to determine if a graph is a graph of a function.



Graph A

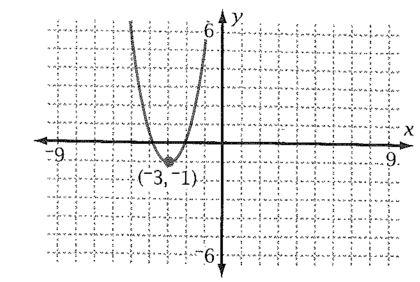
Graph B doesn't qualify as a function because it's possible to find an instance where there is more than one output value for an input value, as shown by the vertical line intersecting the graph in more than one point. The graph fails the vertical line test. The graph still qualifies as a relation, however. Every graph is a **relation**, or correspondence, between variables.



Graph B

The **domain of a function** is the set of allowable input values for the independent variable, while the **range of a function** is the set of resulting output values. The domain of the function described by the graph at the top of page 239 is all real numbers, and the range is real numbers greater than or equal to $^-1$.

The notation $f(x)$ has been used to identify a relation as a function. It offers an easy and standard way of identifying points and indicating transformations. For example, the graph of $y = 2(x + 3)^2 - 1$ is a function, and the equation can be written in function form like this: $f(x) = 2(x + 3)^2 - 1$. The graph is a parabola with the **vertex** at $(^-3, ^-1)$ and a **line of symmetry** at $x = {}^-3$. The graph is congruent to the graph of $y = 2x^2$ (a stretched version of $y = x^2$). The notation $f(^-2)$ represents the y-coordinate when x is $^-2$. So $f(^-2) = 1$.

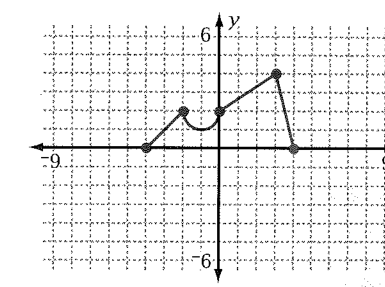


Translations, reflections, stretches or compressions of graphs, and combinations of these, all qualify as transformations. Translations and reflections produce images that remain congruent to the original image. However, with stretches and compressions (like the vertical distortion caused by $af(x)$) the original graph and the new graph are *not* congruent.

In many of the following problems, you will combine translations with stretches or compressions. At times the order won't make any difference. You will always be correct, however, if you perform any stretches *before* you translate points vertically.

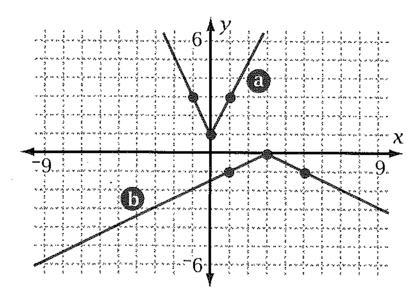
Problem Set 5.8

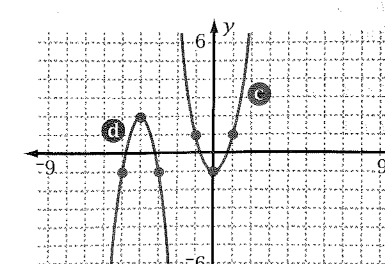
1. Suppose the function pictured is $f(x)$.
   a. $f(3) = $ -?-
   b. $f(^-2) = $ -?-
   c. When is $f(x) = 0$?
   d. When is $f(x) = 2$?
   e. What is the range, R_f, of f?
   f. What is the domain, D_f, of f?

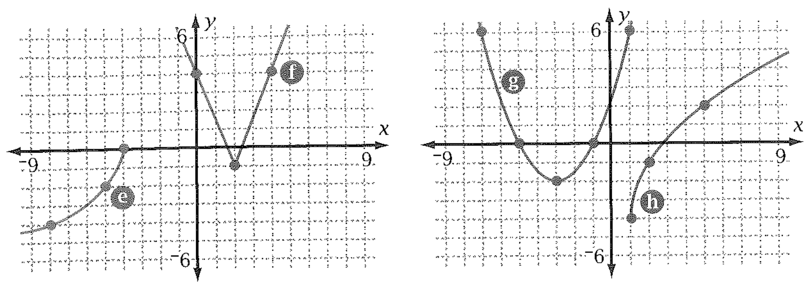


2. Using the graph in Problem 1 as function f, carefully sketch a separate graph of each transformation.
   a. $f(x) - 3$
   b. $f(x - 3)$
   c. $^-f(x)$
   d. $2f(x) - 3$
   e. $f(^-x)$
   f. $f\left(\frac{x}{2}\right)$

3. Use what you know about translations and stretches to write an equation for each graph a–h. Then check your answer by graphing each equation on your calculator.







4. Respond to each of the following with a selection from {all, some, none}. If your choice is some or none, then draw an instance where the graph isn't a function.
   a. Circles are functions.
   b. Parabolas are functions.
   c. Lines are functions.

5. Suppose $f(x) = x^2$. Name a sequence of transformations, in a correct order, that will change $f(x)$ into each equation below.
   a. $y = 2x^2 - 3$ b. $y = (x - 4)^2 - 2$
   c. $y = {}^-(x + 3)^2 + 1$ d. $y = 0.5(x - 2)^2 - 3$

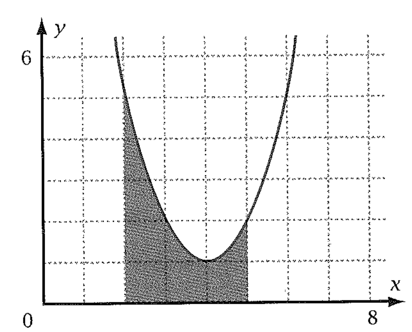
6. Rewrite each equation without parentheses. Graph both forms of the equation to check your work.
   a. $y = 2(x - 4)^2 + 1$
   b. $y = {}^-(x + 3)^2 + 2$
   c. $y = 0.5(x - 2)^2 - 3$

7. In previous problems you found the average value of a function. The value of a function at any point can be thought of as the height of the graph above the x-axis. Therefore, the average value can be considered the average height of the graph. Draw the graph of $y = {}^-(x - 3)^2 + 4$ for $x = 1$ to $x = 5$. (See **Calculator Note 5D**.)

NOTE **5D**

   a. Calculate the average value of the function over this interval. Evaluate the function at each endpoint, and at every 0.2 units in between. Average these values.
   b. Calculate the average value again using values of x every 0.1 unit.
   c. What do you think the average value would be if you used extremely closely spaced x-values? Support your answer.
   d. Draw a rectangle using the interval on the x-axis as the base, and the average height of the function as the height of the rectangle. What is the area of this rectangle?
   e. How do you think this compares with the area enclosed by the curve? Explain.

8. Describe a procedure to find the area indicated in the graph at the right.



9. The Canz4U Container Corporation receives 450 drums of plastic packing pellets every 30 days. The inventory function (drums on hand as a function of days) is
   $I(d) = 450 - \frac{d^2}{2}$.
   Find the average daily inventory. If the cost of keeping one drum is two cents per day, find the average daily holding cost.

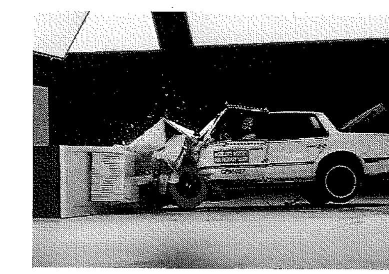
10. Solve each equation for y, and draw the graph.
    a. $2x - 3y = {}^-12$ b. ${}^-2(x + 1.5) + 3(y - 2) - 3 = 0$
    c. $\frac{y}{2} = (x - 3)^2 - 2$ d. ${}^-y + \frac{(x - 3)^2}{2} = 1$

11. Suppose f is a linear function. What is its equation if $f(2) = 6$ and $f({}^-3) = {}^-4$?

12. The distances needed to stop a car on dry pavement in a minimum length of time for various speeds are shown in the table below. Reaction time is considered to be 0.75 sec.

Speed (mi/hr)	10	20	30	40	50	60	70
Stopping distance (ft)	19	42	73	116	173	248	343

   a. Construct a scatter plot for this data.
   b. Use guess-and-check to find the equation of a parabola that "best" fits the points; graph it.
   c. Find the residual sum for this equation.
   d. Predict the stopping distance from 56.5 mi/hr.
   e. How fast are you traveling if you need a stopping distance of 385 ft?



13. You have studied several families of functions (parabolas, square roots, absolute value, semicircles) and transformations of these functions. Discuss the similarities and differences of these transformations among the families.